MR2174099 (2006h:20030) 20E05 (11R52 20F67)

Rattaggi, Diego (CH-GENV-SM)

Anti-tori in square complex groups. (English summary)

Geom. Dedicata **114** (2005), 189–207.

An anti-torus is a (sub)group generated by two elements with no commuting powers. The author analyzes anti-tori in square complex groups, roughly speaking, groups acting on a product of two trees, and more especially in commutative transitive square complex groups where a dichotomy exists: either two elements generate an anti-torus or they commute. These results are applied to the groups $\Gamma_{p,l}$ defined using quaternion algebras and first studied by S. Mozes [in *Symbolic dynamics and its applications (New Haven, CT, 1991)*, 319–325, Contemp. Math., 135, Amer. Math. Soc., Providence, RI, 1992; MR1185097 (93j:28032)]. Then the author addresses the problem of finding free subgroups in $\Gamma_{p,l}$ and $\mathrm{SO}_3(\mathbf{Q})$ and gives a criterion for determining that a certain subgroup of a square complex group is not free.

Reviewed by *Olivier Guichard*

References

1. Bestvina, M.: Questions in geometric group theory, version of 22 August 2000, http://www.math.utah.edu/∼bestvina
2. Bridson, M. R. and Haefliger, A.: *Metric Spaces of Non-positive Curvature*, Grundle. Math. Wiss. 319, Springer-Verlag, Berlin, 1999. MR1744486 (2000k:53038)
3. Bridson, M. R. and Wise, D. T.: $\mathcal{VH}$ complexes, towers and subgroups of $F \times F$, *Math. Proc. Cambridge Philos. Soc.* **126** (3) (1999), 481–497. MR1684244 (2000c:20050)
4. Burger, M. and Mozes, S.: Groups acting on trees: from local to global structure, *Inst. Hautes Études Sci. Publ. Math.* **92** (2000), 113–150 (2001). MR1839488 (2002i:20041)
5. Burger, M. and Mozes, S.: Lattices in product of trees, *Inst. Hautes Études Sci. Publ. Math.* **92** (2000), 151–194 (2001). MR1839489 (2002i:20042)
6. Burger, M., Mozes, S. and Zimmer, R. J.: Linear representations and arithmeticity for lattices in products of trees, Preprint, 2004, http://www.fim.math.ethz.ch/preprints/2004/burger-mozes-zimmer.pdf
7. Davidoff, G., Sarnak, P. and Valette, A.: *Elementary Number Theory, Group Theory, and Ramanujan Graphs*, London Math. Soc. Student Texts 55, Cambridge University Press, Cambridge, 2003. MR1989434 (2004f:11001)
8. The GAP group, Aachen, St. Andrews, GAP - Groups, Algorithms, and Programming, Version 4.2; 2000, http://www.gap-system.org
9. Kimberley, J. S. and Robertson, G.: Groups acting on products of trees, tiling systems and analytic K-theory, *New York J. Math.* **8** (2002), 111–131 (electronic). MR1923572 (2003j:20042)
10. Liu, G. and Robertson, L. C.: Free subgroups of $\mathrm{SO}_3(\mathbb{Q})$, *Comm. Algebra* **27** (4) (1999), 1555–1570. MR1679704 (99m:20048)

11. Lubotzky, A.: *Discrete Groups, Expanding Graphs and Invariant Measures*, with an appendix by Jonathan D. Rogawski, Prog. in Math. 125, Birkhäuser, Basel, 1994. MR1308046 (96g:22018)
12. Margulis, G. A.: *Discrete Subgroups of Semisimple Lie Groups*, Springer-Verlag, New York, 1991. MR1090825 (92h:22021)
13. Mozes, S.: *A Zero Entropy, Mixing of all Orders Tiling System*, Symbolic dynamics and its applications, New Haven, CT, 1991, 319–325, *Contemp. Math.* 135, Amer. Math. Soc., Providence, RI, 1992. MR1185097 (93j:28032)
14. Mozes, S.: On closures of orbits and arithmetic of quaternions, *Israel J. Math.* **86**(1–3) (1994), 195–209. MR1276134 (95k:22014)
15. Mozes, S.: Actions of Cartan subgroups, *Israel J. Math.* **90** (1–3) (1995), 253–294. MR1336326 (97b:22022)
16. Rattaggi, D.: Computations in groups acting on a product of trees: normal subgroup structures and quaternion lattices, PhD thesis, ETH Zürich, 2004.
17. Rattaggi, D. and Robertson, G.: Abelian subgroup structure of square complex groups and arithmetic of quaternions, *J. Algebra* **286** (1) (2005), 57–68. MR2124808 (2005i:22010)
18. Stallings, J. R.: On torsion-free groups with infinitely many ends, *Ann. of Math.* **88** (2) (1968), 312–334. MR0228573 (37 #4153)
19. Wagon, S.: *The Banach-Tarski Paradox*, with a foreword by Jan Mycielski, Corrected reprint of the 1985 original, Cambridge University Press, Cambridge, 1993. MR0803509 (87e:04007)
20. Wise, D. T.: Non-residually curved squared complexes, aperiodic tilings, and nonresidually finite groups, PhD thesis, Princeton University, 1996.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.