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Periodic automorphisms of surfaces: invariant circles and maximal orders. (English. English summary)

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W. H. Meeks, III [J. Differential Geom. **14** (1979), no. 3, 377–383 (1980); MR0594708 (83e:57003)] considered orientation-preserving diffeomorphisms of finite order on a compact orientable surface F_g of genus g. He showed that if $g \leq 10$, then the diffeomorphism has an invariant circle. If $g = 11$, then there need not be an invariant circle; nevertheless, in some sense, "most" periodic orientation-preserving diffeomorphisms $f\colon F_g \to F_g$ have an invariant circle; i.e., a circle C embedded in F_g such that $f(C) = C$. This led Meeks to conjecture that for an infinite number of g, every such diffeomorphism must have an invariant circle.

The authors use Meeks' own technique to do a numerical search and give evidence that Meeks' conjecture may be false. They show that for exactly 43 values of g, $0 \leq g \leq 105$, every orientation-preserving diffeomorphism of finite order has an invariant circle, but for $106 \leq g \leq 10\,000$, this is not the case.

The authors also consider the existence of invariant essential circles as well as nonorientable surfaces. *David G. Wright* (1-BYU)

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